

General Certificate of Education (A-level) January 2012

Mathematics

MPC1

(Specification 6360)

Pure Core 1

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
−x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

MPCI		1		
Q	Solution	Marks	Total	Comments
1(a)	$(OA^2 =) 6^2 + (-4)^2 ; (OB^2 =) (-2)^2 + 7^2$	M1		either correct PI by 52 or 53 seen
	$(OA^2 =) 52$ and $(OB^2 =) 53$ or $(OA =) \sqrt{52}$ and $(OB =) \sqrt{53}$	A1		both correct values 52 or $\sqrt{52}$ and 53 or $\sqrt{53}$ seen
	$OA = \sqrt{52}$ and $OB = \sqrt{53}$ $\Rightarrow OA < OB$	A1	3	or $OA^2 = 52$ and $OB^2 = 53$ correct working + concluding statement involving OA and/or OB
(b)(i)	$grad AB = \frac{7+4}{-2-6}$	M1		condone one sign error
	$=-\frac{11}{8}$	A1	2	
(ii)	y4 = 'their grad AB'(x - 6) or $y - 7 = 'their grad AB'(x - 2)$	M1		or $y =$ 'their grad AB ' $x + c$ and attempt to find c using $x = 6$, $y = -4$ or $x = -2$, $y = 7$
	$y+4=-\frac{11}{8}(x-6)$ OE	A1		any correct form eg $y = -\frac{11}{8}x + \frac{34}{8}$ but must simplify $$ to $+$
	$\Rightarrow 11x + 8y = 34$	A1	3	condone $8y + 11x = 34$ or any multiple of these equations
(c)	$(\operatorname{grad} AC =) \frac{8}{11}$	B1√		FT -1 / 'their grad AB'
	$\frac{4}{k-6} = 'their \frac{8}{11}' \text{ OE}$ $\Rightarrow 2k-12=11$	M1		equating gradients; LHS must be correct and RHS is "attempt" at perp grad to AB
	$\Rightarrow k = \frac{23}{2}$	A1cso	3	k = 11.5 OE
	Total		11	

Total 11

(c) Alternative: Eqn $AC: (y+4) = 'their \frac{8}{11}' (x-6) B1 \checkmark (11y=8x-92)$ AND must sub y=0 for M1

or $(y-0) = 'their \frac{8}{11}' (x-k) B1 \checkmark$ AND must sub x=6, y=-4 for M1

Q	Solution	Marks	Total	Comments
2(a)	(x-6)(x+2)	B1	1	ISW for $x = 6$, $x = -2$ etc
(b)	$ \begin{array}{c c} x & -2 \\ x & -6 \end{array} $	B1√		correct x values or FT 'their' factors (x-intercepts stated or marked on sketch) may be seen in (a)
	y = -12	B1		(stated <i>or</i> –12 marked on sketch)
	∪ – shaped curve	M1		approximately
	"correct" shape in all 4 quadrants with minimum to right of <i>y</i> -axis	A1	4	
(c)(i)	$(x-2)^2$	M1		p=2
	$(x-2)^2-16$	A1	2	p=2 and $q=16$
(ii)	(Minimum value is) -16	B1√	1	FT 'their $-q$ '
(d)	Replacing each x by $x + 3$ OR adding 2 to their quadratic	M1		in original equation or 'their' completed square or factorised form or replacing y by $y-2$
	$y = \left[(x+3)^2 - 4(x+3) - 12 \right] + 2$ or $y = (x+1)^2 - 14$ or $y = x^2 + 2x - 13$ or $y - 2 = (x-3)(x+5)$ Total	A1	2	OE any correct equation in x and y unsimplified

Q Q	Solution	Marks	Total	Comments
3(a)(i)	$\left(3\sqrt{2}\right)^2 = 18$	B1	1	
(ii)	$(3\sqrt{2} - 1)^{2} = 'their 18' - 3\sqrt{2} - 3\sqrt{2} + 1$ $= 18 - 3\sqrt{2} - 3\sqrt{2} + 1$	M1		FT their $(3\sqrt{2})^2$
	$= 18 - 3\sqrt{2} - 3\sqrt{2} + 1$	A1		FT their $(3\sqrt{2})^2$ $(=19-6\sqrt{2})$ $(=11+6\sqrt{2})$
	$\left(3+\sqrt{2}\right)^2 = 9+3\sqrt{2}+3\sqrt{2}+2$	B1		$\left(=11+6\sqrt{2}\right)$
	\Rightarrow Sum = 30	A1cso	4	
(b)	$\frac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}} \times \frac{2\sqrt{5} - \sqrt{2}}{2\sqrt{5} - \sqrt{2}}$	M1		
	Numerator = $8(\sqrt{5})^2 - 4\sqrt{5}\sqrt{2} - 14\sqrt{5}\sqrt{2} + 7(\sqrt{2})^2$	m1		correct unsimplified $(=54-18\sqrt{10})$
	Denominator = $(2\sqrt{5})^2 - (\sqrt{2})^2$ = 18	B1		must be seen as denominator
	\Rightarrow Answer = $3 - \sqrt{10}$	Alcso	4	
	Total		9	

O O	Solution	Marks	Total	Comments
		M1		one term correct
4 (a)(i)	$\left(\frac{dy}{dx} = \right) 5x^4 - 6x + 1$	A1		another term correct
	(dx)	A1	3	all correct (no + c etc)
	$\left(\frac{dy}{dx} = \right) 5x^4 - 6x + 1$ $\left(\frac{d^2y}{dx^2} = \right) 20x^3 - 6$	B1√	1	FT 'their' $\frac{dy}{dx}$
(b)	$x = -1 \Rightarrow \frac{dy}{dx} = 5(-1)^4 - 6(-1) + 1 (= 12)$ $\Rightarrow y = 12(x+1)$	M1		must sub $x = -1$ into 'their' $\frac{dy}{dx}$
	dx			$\mathbf{d}x$
	$\Rightarrow y = 12(x+1)$	A1cso	2	any correct form with $(x-1)$ simplified
				condone $y = 12x + c$, $c = 12$
(c)	$x=1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 5 - 6 + 1$	M1		sub $x = 1$ into their $\frac{dy}{dx}$
	dy			u.
	$\frac{dy}{dx} = 0 \implies$ stationary point	Alcso		shown = 0 plus correct statement
	when $x = 1$, $\frac{d^2 y}{dx^2} = 14$ $\Rightarrow (B \text{ is a }) \text{ minimum (point)}$	E1	3	or $\frac{d^2 y}{dx^2} = 20 - 6 > 0$ $\Rightarrow (B \text{ is a) minimum (point)}$ must have correct $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$ for E1
(3)(2)	$x^6 - 3x^3 - x^2 - 5$	M1		one term correct
(a)(1)	$\frac{x^6}{6} - \frac{3x^3}{3} + \frac{x^2}{2} + 5x$	A1 A1		another term correct all correct (may have $+ c$)
		AI		all correct (may have + t)
	$\left[\frac{1}{6} - 1 + \frac{1}{2} + 5\right] - \left[\frac{1}{6} + 1 + \frac{1}{2} - 5\right]$	m1		'their' $F(1) - F(-1)$ with powers of 1 and -1 evaluated correctly
	= 8	Alcso	5	
(ii)	'their answer to part (i)' -2	M1		
	\Rightarrow Area = 6	A1cso	2	
	Total		16	

MPC1 (cont)		M	7F 4 1	
Q	Solution	Marks	Total	Comments
5(a)	$p(-2) = (-2)^3 + (-2)^2 c + (-2)d - 12$	M1		p(-2) attempted or
				long division by $x+2$ as far as remainder
	'their' $-8 + 4c - 2d - 12 = -150$	m1		putting expression for remainder = -150
	$\Rightarrow 2c - d + 65 = 0$	A1cso	3	AG terms all on one side in any order (check that there are no errors in working)
(b)	$p(3) = 3^3 + 3^2 c + 3d - 12$	M1		p(3) attempted or long division by x -3 as far as remainder
	9c + 3d + 15 = 0	A1	2	any correct equation with terms collected eg $3c+d=-5$
(c)	$ 2c - d + 65 = 0 $ $3c + d + 5 = 0 $ $\Rightarrow 5c = -70$	M1		Elimination of c or d
	$\Rightarrow c = -14$, $d = 37$ OE	A1		value of <i>c</i> or <i>d</i> correct unsimplified
	\Rightarrow c = 14, u = 37 OL	A1	3	both c and d correct unsimplified
	Total		8	
6(a)	Sides are x and $x + 4$			
	$\Rightarrow x+x+x+4+x+4>30$			
	or 2x + 2x + 8 > 30			
	or $2(2x+4) > 30$			must see this line OE
	or $4x + 8 > 30$			
	$(\Rightarrow 4x > 22)$			
	$\Rightarrow 2x > 11$	B1	1	AG (be convinced) condone $11 < 2x$
(I)	(, , 1) , , 00			41.11.05
(b)	$x\left(x+4\right) < 96$			must see this line OE
	$\Rightarrow x^2 + 4x - 96 < 0$	B1	1	AG
				and fortune on a second second second
(c)	(x+12)(x-8)	M1		correct factors or correct quadratic equation formula
	Critical values 8, -12	A1		oquation formula
		711		
	$ \begin{array}{c cccc} & y & & \text{or} \\ \hline & -12 & & & \\ \hline & & & \\$	M1		sketch or sign diagram
	→ 12 × × × °	A 1	4	accept x < 8 AND x > 12
	$\Rightarrow -12 < x < 8$	Alcso	4	accept $x < 8$ AND $x > -12$ but not $x < 8$ OR $x > -12$
				nor $x < 8$, $x > -12$
	1			
(d)	$5\frac{1}{2} < x < 8$	B1	1	
	Total		7	
1	10tai		,	1

Q Q	Solution	Marks	Total	Comments
7(a)	$(x+7)^2 + (y-5)^2$	M1		one term correct; condone $(x-7)^2$
		A1		both terms correct with squares
				and plus sign between terms
	$(x+7)^2 + (y-5)^2 = 5^2$	A1cao	3	condone 25 for 5 ²
(b)(i)	C(-7,5)	B1√		correct or FT 'their' circle equation
(ii)	<i>r</i> = 5	B1√	2	correct or FT 'their' $r^2 > 0$ condone $\sqrt{25}$ etc but not $\pm \sqrt{25}$
(c)	must draw axes	M1		freehand circle with C correct or FT 'their C' for quadrant of centre
	-7	A1	2	circle touching x-axis at -7 with -7 marked (need not show 5 on y-axis) but circle must not touch y-axis
(d)(i)	$x^{2} + (kx+6)^{2} + 14x - 10(kx+6) + 49 = 0$			clear attempt to sub $y = kx + 6$ into original or 'their' circle equation
	$x^{2} + k^{2}x^{2} + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$	M1		and attempt to multiply out
	$(1+k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2+1)x^2 + 2(k+7)x + 25 = 0$	Alcso	2	AG condone $x^2(1+k^2) + 2x(7+k) +$ etc
(ii)	Equal roots ' $b^2 - 4ac = 0$ '	B1		allow statement alone if discriminant in terms of k attempted
	$\left[2(k+7)\right]^2-4\times25(k^2+1)$	M1		discriminant (condone one slip)
	$4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$			
	$-24k^2 + 14k + 24 = 0$			
	$\Rightarrow 12k^2 - 7k - 12 = 0$	A1	3	AG all working correct but = 0 must appear before last line
(iii)	(4k+3)(3k-4)	M1		correct factors or correct use of
				formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$
	$\Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3}$ OE	A1	2	
	are values of k for which line is a tangent			
	Total		14	
	TOTAL		75	